Corrigé Examen EM
L2 - Physique - juin 2010

II) 1) $\Phi_1 = N_1 \iint_{S_1} \vec{B}_1 . \vec{n}_1 \, dS = N_1 \mu_0 n_1 I_1 \iint_{S_1} \vec{e}_z . \vec{e}_z \, d\rho \, \rho \, d\varphi$

$$\Phi_1 = N_1 \mu_0 \frac{N_1}{\ell_1} I_1 \pi R_1^2 = L_1 I_1$$

$$L_1 = \frac{N_1^2}{\ell_1} \mu_0 \pi R_1^2 = n_1^2 \ell_1 \mu_0 \pi R_1^2 \simeq \mu_0 n_1 \ell_1 \pi R^2$$

2) $\Phi_{\vec{B}_1 \to C_2} = \frac{N_2}{\ell_2} \times z \iint_{S_2} \vec{B}_1 . \vec{n}_2 \, dS = n_2 n_1 \mu_0 z I_1 \pi R^2$

$$= M I_1 \implies M = n_1 n_2 \mu_0 z \pi R^2$$

3) $$K = \frac{n_1 n_2 \mu_0 \pi R^2 z}{\sqrt{L_1 L_2}} = \frac{n_1 n_2 \mu_0 \pi R^2 z}{\left(\mu_0 n_1^2 \ell_1 \pi R^2 \mu_0 n_2^2 \ell_2 \pi R^2\right)^{1/2}}$$

$$K = \frac{z}{\sqrt{\ell_1 \ell_2}} \quad \text{(Sans dimension)}$$

$$\begin{cases} z = 0 \longrightarrow K = 0 \\ z = \ell_1 \longrightarrow K = \sqrt{\frac{\ell_1}{\ell_2}} \end{cases} \qquad 0 \leq K \leq \sqrt{\frac{\ell_1}{\ell_2}}$$

4) $\vec{B}_1 = \mu_0 n_1 I_1 \vec{e}_z \qquad \vec{B}_2 = \mu_0 n_2 I_2 \vec{e}_z$

$$\vec{B}_3 = \vec{B}_1 + \vec{B}_2 = \mu_0 \left(n_1 I_1 + n_2 I_2\right) \vec{e}_z$$

$$e_m = \frac{\vec{B}_1^2}{2\mu_0} + \frac{\vec{B}_2^2}{2\mu_0} + \frac{1}{2\mu_0} \vec{B}_3^2$$

$$\mathcal{E}_m = \frac{\mu_0 n_1^2 I_1^2}{2\mu_0}\iiint \rho\, d\rho\, d\varphi\, dz = \frac{\mu_0 n_1^2 I_1^2}{2}\pi R^2 \int_z^{l_1} dz' +$$

$$+ \frac{\mu_0 n_2^2 I_2^2}{2\mu_0}\iiint \rho\, d\rho\, d\varphi\, dz = \frac{\mu_0 n_2^2 I_2^2 \pi R^2}{2}\int_{-l_2+z}^{0} dz' +$$

$$+ \frac{\mu_0^2 (n_1 I_1 + n_2 I_2)^2}{2\mu_0}\iiint \rho\, d\rho\, d\varphi\, dz = \frac{\mu_0 (n_1 I_1 + n_2 I_2)^2}{2}\pi R^2 \int_0^z dz'$$

$$\mathcal{E}_m = \frac{\mu_0 n_1^2 I_1^2 \pi R^2}{2}(l_1 - z) + \frac{\mu_0 n_2^2 I_2^2 \pi R^2}{2}(l_2 - z)$$

$$+ \frac{\mu_0 n_1^2 I_1^2 \pi R^2 z}{2} + \mu_0 n_1 n_2 I_1 I_2 \pi R^2 z$$

$$+ \frac{\mu_0 n_2^2 I_2^2 \pi R^2 z}{2}$$

$$\mathcal{E}_m = \frac{\mu_0 n_1^2 I_1^2 \pi R^2 l_1}{2} + \frac{\mu_0 n_2^2 I_2^2 \pi R^2 l_2}{2}$$

$$+ \mu_0 n_1 n_2 I_1 I_2 \pi R^2 z$$

$$\mathcal{E}_m = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

$$\Longrightarrow \boxed{M = \mu_0 n_1 n_2 \pi R^2 z}$$